

# TEACHING STRATEGIES FOR MENTAL MATHEMATICS (INTERMEDIATE PHASE)

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Data Mind

*Mental mathematics is one of the most important tools for learning mathematics. It not only means to calculate quickly, but involves conceptual understanding and problem solving. Mental mathematics is also a very useful life skill. It is also a very important component of the NCS Curriculum and Assessment Policy for Mathematics. The CAPS document lists the bonds, number facts and tables that Intermediate Phase learners are expected to know and recall for each grade. However, to improve mental calculations, learners need to be taught the most efficient strategies explicitly. This workshop aims to revise the strategies for calculation suggested in the CAPS document and propose some activities that can be used to practise these strategies.*

## **RAPID RECALL CALCULATION FACTS**

Tables of the calculation facts for each grade are given below. The list is not exhaustive but includes the key strategies that can be used when developing mental mathematics skills. Each year is based on the previous year and teachers should ensure that the knowledge from previous years is revised and carried forward.

This table has been based on the NCS Curriculum and Assessment Policy Statement for Intermediate Phase Mathematics.

## Grade 4

Grade 4 learners should have	Mental Strategies
<p><b>Rapid recall of number facts for</b></p> <ul style="list-style-type: none"> <li>• number bonds: addition and subtraction facts for:               <ul style="list-style-type: none"> <li>◊ units</li> <li>◊ multiples of 10, 100 and 1 000</li> </ul> </li> <li>• times tables involving multiplication of whole numbers to at least 10 x 10</li> </ul> <p><b>Rapid recall of multiplication facts for</b></p> <ul style="list-style-type: none"> <li>• units by multiples of 10</li> <li>• units by multiples of 100</li> </ul> <p><b>Number range for multiples and factors</b></p> <p>Multiples of 1-digit numbers to at least 100</p>	<p><b>Calculation techniques</b></p> <ul style="list-style-type: none"> <li>• count forwards and backwards in 2s, 3s, 5s, 10s, 25s, 50s, between 0 and at least 10 000</li> <li>• count forwards and backwards in 100s between 0 and at least 1 000</li> <li>• Ordering and comparing up to 4-digit numbers</li> <li>• Represent odd and even numbers to at least 1 000</li> <li>• doubling and halving,</li> <li>• using a number line,</li> <li>• using multiplication to do division,</li> <li>• multiplying by 10 and 100</li> <li>• multiplying by multiples of 10 and 100</li> <li>• dividing by 10, 100 and 1 000</li> <li>• recognize the place value of digits in whole numbers to at least 4-digit numbers</li> <li>• Round off to the nearest 10, 100, 1 000</li> <li>• estimation</li> <li>• rounding off to the nearest 10 and compensating</li> <li>• building up and breaking down numbers</li> <li>• adding and subtracting units, multiples of 10 and multiples of 100 to/from any 3-digit number</li> <li>• using the inverse relationship between addition and subtraction</li> <li>• using multiplication and division as inverse operations.</li> <li>• Recognize and use the commutative; associative; and distributive properties of whole numbers.</li> </ul>

Table 1: Summary of Grade 4 mental mathematics facts and mental strategies

## Grade 5

Grade 5 learners should have	Mental Strategies
<p><b>Rapid recall of number facts for</b></p> <ul style="list-style-type: none"> <li>• number bonds: addition and subtraction facts for:               <ul style="list-style-type: none"> <li>◊ units</li> <li>◊ multiples of 10, 100, 1 000 and 10 000</li> </ul> </li> <li>• times tables involving multiplication of whole numbers to at least 10 x 10</li> </ul> <p><b>Rapid recall of multiplication facts for</b></p> <ul style="list-style-type: none"> <li>• units by multiples of 10, 100, 1 000, 10 000</li> </ul> <p><b>Number range for multiples and factors</b></p> <p>Multiples of 2-digit whole numbers to at least 100</p> <p>Factors of 2-digit whole numbers to at least 100</p>	<p><b>Calculation techniques</b></p> <ul style="list-style-type: none"> <li>• count forwards and backwards in whole number intervals up to at least 10 000</li> <li>• Ordering and comparing up to 6-digit numbers</li> <li>• Represent odd and even numbers to at least 1 000</li> <li>• multiples</li> <li>• factors</li> <li>• doubling and halving,</li> <li>• using a number line,</li> <li>• using multiplication to do division,</li> <li>• multiplying by 10, 100 and 1 000</li> <li>• multiplying by multiples of 10 and 100</li> <li>• dividing by 10, 100 and 1 000</li> <li>• recognize the place value of digits in whole numbers to at least 6-digit numbers</li> <li>• Round off to the nearest 5, 10, 100, 1 000</li> <li>• estimation</li> <li>• rounding off to the nearest 10, 100 and 1 000 and compensating</li> <li>• building up and breaking down numbers</li> <li>• adding and subtracting units, multiples of 10, 100 and 1 000 to/from any 5-digit number</li> <li>• using the inverse relationship between addition and subtraction</li> <li>• using multiplication and division as inverse operations.</li> <li>• Recognize and use the commutative; associative; and distributive properties of whole numbers.</li> <li>• adding and subtracting in columns</li> <li>• 0 in terms of its additive property</li> <li>• 1 in terms of its multiplicative property</li> </ul>

Table 2: Summary of Grade 5 mental mathematics facts and mental strategies

## Grade 6

Grade 6 learners should have	Mental Strategies
<p><b>Rapid recall of number facts for</b></p> <ul style="list-style-type: none"> <li>number bonds: addition and subtraction facts for:               <ul style="list-style-type: none"> <li>◇ units</li> <li>◇ multiples of 10, 100, 1 000 and 10 000</li> </ul> </li> <li>times tables involving multiplication of whole numbers to at least 12 x 12</li> </ul> <p><b>Rapid recall of multiplication facts for</b></p> <ul style="list-style-type: none"> <li>units by multiples of 10, 100, 1 000, 10 000</li> </ul> <p><b>Number range for multiples and factors</b></p> <p>Multiples of 2- and 3-digit whole numbers to at least 100</p> <p>Factors of 2- and 3-digit whole numbers to at least 100</p> <p>Prime factors of numbers to at least 100</p>	<p><b>Calculation techniques</b></p> <ul style="list-style-type: none"> <li>count forwards and backwards in whole number intervals up to at least 10 000</li> <li>Ordering and comparing up to 9-digit numbers</li> <li>Represent prime numbers to at least 100</li> <li>Represent odd and even numbers to at least 1 000</li> <li>multiples</li> <li>factors</li> <li>doubling and halving,</li> <li>using a number line,</li> <li>using multiplication to do division,</li> <li>multiplying by 10, 100 and 1 000</li> <li>multiplying by multiples of 10 and 100</li> <li>dividing by 10, 100 and 1 000</li> <li>recognize the place value of digits in whole numbers to at least 9-digit numbers</li> <li>Round off to the nearest 5, 10, 100, 1 000, 10 000, 100 000 and 1 000 000</li> <li>estimation</li> <li>rounding off to the nearest 10, 100 and 1 000 and compensating</li> <li>building up and breaking down numbers</li> <li>adding and subtracting units, multiples of 10, 100 and 1 000 to/from any 5-digit number</li> <li>using the inverse relationship between addition and subtraction</li> <li>using multiplication and division as inverse operations.</li> <li>Recognize and use the commutative; associative; and distributive properties of whole numbers.</li> <li>adding, subtracting and multiplying in columns</li> <li>long division</li> <li>0 in terms of its additive property</li> <li>1 in terms of its multiplicative property</li> </ul>

Table 3: Summary of Grade 6 mental mathematics facts and mental strategies

## **CHOOSING STRATEGIES**

It is important to emphasize that

- learners need efficient and quick methods for mental mathematics
- learners should choose a method that is sensible
- teachers consolidate the main features of the strategies used at the end of the session.

## **KEY STRATEGIES AND ACTIVITIES**

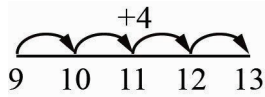
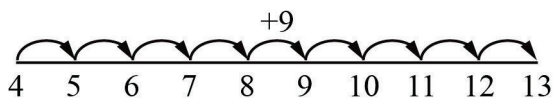
- Counting
- Ordering a given set of numbers
- Adding three or four small numbers by finding pairs that make 10
- Doubling and halving
- Near doubles
- Place value (partitioning using multiples of 10, 100 and 1 000)
- Change a number to a multiple of 10 and then subtract or add 1, 2 or 3 (compensating)
- Building up and breaking down numbers
- Use the relationship between addition and subtraction
- Use the relationship between multiplication and division

### **Counting**

Learners learn to count by beginning at one and then counting on in ones. Although it is important for learners to do this counting as a number-based exercise, it is important that they also be reminded that counting can be object-based and context-based (candles on a cake). Give learners opportunities to count real objects like

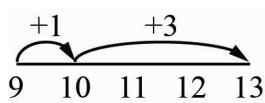
- all the chairs in the classroom
- the number of pencils in a jar
- the number of boys/girls in a group
- the number of shoes in a class and ask them why it may be more useful to count the shoes in twos
- Ask learners to hold one hand in the air with five fingers spread out. What would they count in if they counted all the fingers?
- Ask learners when counting in tens, 50s or 100s would be useful.

Number lines can be useful to help learners count forwards and backwards. Empty number lines can be used for single digit numbers, e.g.  $4 + 9$ .



Encourage learners to see that the second number line may be more efficient than the first. They should start from the bigger number and count on.

Another way of using the empty number line could be to count on one to 10 and then count on another 3.



We can also use counting to find the multiples of a number, for example.

- 1 2; 4; 6; 8; \_\_\_; \_\_\_; \_\_\_
- 2 3; 6; 9; 12; \_\_\_; \_\_\_; \_\_\_
- 3 10; 20; 30; 40; \_\_\_; \_\_\_; \_\_\_
- 4 100; 200; 300; \_\_\_; \_\_\_; \_\_\_; 700

In general, counting helps learners develop strategies for calculating. Learners are calculating when they 'skip count'.

### **Ordering and reordering (commutative property)**

When learners order numbers they use an important aspect of place value. Numbers with more digits will be larger than those with fewer digits. The smallest 2-digit number (10) is larger than the biggest 1-digit number (9) and so on. If the numbers that need to be ordered have the same number of digits, learners start by comparing the digits in each place value starting from the left.

Some concrete practice can be to take 10 to 15 cards with numbers on them. Ask learners to shuffle them and to arrange the cards from smallest to biggest. You can also remove some cards and then ask learners to replace them in order; work out how many cards there are between 63 and 75 and then count the cards between 63 and 75. Are the two numbers the same? Why or why not?

Number lines can also be used to compare numbers. Remember to revise words like ‘before’, ‘after’ and ‘between’.

	Number	comes before	comes after	+1	-1
1	11				
2	199				
3	935				
4	60				
5	174				

Sometimes it is easier to do a calculation by changing the order of the numbers (Commutative Law). For example

Look for pairs of numbers that make 10 or 20 and use these first.

1  $5 + 7 + 3 = 5 + (\underline{\quad} + 3) = \underline{\quad}$

2  $4 + 7 + 6 = (4 + 6) + 7 = \underline{\quad}$

3  $3 + 18 + 2 = 3 + (18 + \underline{\quad}) = \underline{\quad}$

It is important that learners realise that they can change the order of the numbers when there are just addition signs, but it gets more complicated when there are minus signs. Order does matter in subtraction!

$9 - 7$  does not equal  $7 - 9$ .

When there is more than one subtraction, the order can be changed. Teachers should judge the confidence of their group before introducing this technique.

$$17 - 5 - 8 = 17 - 8 - 5$$

### **Doubling and halving**

Practice doubles with learners often so that they attain instant recall of them. Learners often find the doubles the easiest facts to remember. They can be used to:

- simplify calculations
- double one number and halve the other in a product.

Double	8	12	15	16	22	50
Halve	20	24	18	16	15	90

## Near doubles

Doubles can then be used to add numbers that are close to doubles. For example, learners know that  $18 + 18 = 36$ , so they can be encouraged to see that  $18 + 19$  will be one more than 36.

As learners become more confident, give them numbers that are two and then three apart to use to practice this strategy.

- 1  $3 + 4 = (3 + 3) + 1 = \underline{\quad}$
- 2  $16 + 15 = (15 + 15) + 1 = \underline{\quad}$
- 3  $19 + 20 = (19 + \underline{\quad}) + 1 = \underline{\quad}$
- 4  $30 + 29 = (30 + \underline{\quad}) - \underline{\quad} = \underline{\quad}$
- 5  $100 + 102 = (100 + \underline{\quad}) + \underline{\quad} = \underline{\quad}$

	Number	Double	Double + 1	Double + 2
1	4	$4 + 4 = 8$	$4 + 5 = 9$	$4 + 6 = 10$
2	12	$12 + 12 = \underline{\quad}$	$12 + \underline{\quad} = \underline{\quad}$	$12 + 14 = \underline{\quad}$
3	25			

## Place value

Learners should be able to break down numbers like  $957 = 900 + 50 + 7$ .

		1 000	100	10	1
1	542		5	4	2
2	2 118				
3	6 261				

This is a useful strategy for adding and subtracting. Both numbers can be broken down like this, but it may be quicker to just break one of the numbers.

### Example: $7 + 32$

Think	Do
Leave the first number and break down the second number.	$7 + (30 + 2)$
Add $7 + 30$ and count on by 6	$37 + 2 = 39$



**Example:**  $48 - 30$

<b>Think</b>	<b>Do</b>
Break down the number that has tens and units	$40 + 8 - 30$
Rearrange.	$40 - 30 + 8$
Subtract 30 and count on 8	$10 + 8 = 18$

Break down the numbers and add or subtract.

1  $32 + 47 = (30 + 2) + (40 + 7) = \underline{\quad}$

2  $19 + 15 = (\underline{\quad} + \underline{\quad}) + (10 + 5) = \underline{\quad}$

3  $99 - 84 = (90 + \underline{\quad}) - (80 + \underline{\quad}) = \underline{\quad}$

**Change a number to a multiple of 10 and then subtract or add 1, 2 or 3 (compensating)**

It is quite useful when doing mental mathematics to be able to recognise that one of the numbers is close to 10 or a multiple of 10. The other number is then used to make up the 10 by breaking it down.

$$17 + 8 = 17 + 3 + 5 = 20 + 5 = 25$$

$$57 + 15 = 57 + 3 + 12 = \underline{\quad} + 12 = \underline{\quad}$$

$$48 - 24 = \underline{\quad} - 8 - 16 = \underline{\quad}$$

$$5,9 + 2,4 = \underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

This is similar to the way change used to be counted back. If the item cost R37 and you paid with a R50 note, the change would start at R37 and then  $R3 + R10 = R13$  would be added to get R50.

**Use the relationship between addition and subtraction**

Every addition calculation can be replaced by an equivalent subtraction calculation and similarly every subtraction can be replaced by an addition. For example with addition,

$$17 + 12 = 29$$

$$17 = 29 - 12$$

$$12 = 29 - 17$$

For subtraction,

$$38 - 16 = 22$$

$$38 = 16 + 22$$

$$16 = 38 - 22$$

1  $7 + 9 = \underline{\quad}$  and  $16 - 9 = \underline{\quad}$  and  $\underline{\quad} - 7 = 9$

2  $19 + 3 = \underline{\quad}$  and  $\underline{\quad} - 19 = \underline{\quad}$  and  $\underline{\quad} - 3 = 19$

3  $\underline{\quad} + 4 = 67$  and  $67 - 4 = \underline{\quad}$  and  $67 - \underline{\quad} = 4$

4  $27 - \underline{\quad} = 13$  and  $13 + 14 = \underline{\quad}$  and  $\underline{\quad} - 13 = 14$

### **Multiplication and division**

One of the key factors in learners developing mental mathematics skills is the instant recall of multiplication and division facts. In the Intermediate Phase, the Grade 4 and 5 learners should know their multiplication and division tables up to  $10 \times 10$  and in Grade 6 to  $12 \times 12$ . This takes time and practice.

Skip counting can be used to work out that  $5 \times 3 \rightarrow 3; 6; 9; 12; \underline{15}$

The commutative property makes multiplying  $25 \times 4$  easier by saying  $4 \times 25 \rightarrow 25; 50; 75; 100$ .

Learners can also reorder numbers to calculate 'easier' facts first, for example:

$$4 \times 6 \times 5 = (4 \times 5) \times 6 = 20 \times 6 = 120$$

$$12 \times 9 = 12 \times 10 - 12 = 120 - 12 = 108$$

Doubling and halving can be used to find easier numbers to work with, for example  $16 \times 5 = 8 \times 10 = 80$  (halve 16 and double 5)

Every multiplication calculation can be replaced by an equivalent division calculation and similarly every division can be replaced by a multiplication.

$$30 \times 7 = \underline{\quad}$$

$$\underline{\quad} \div 7 = 30$$

$$\underline{\quad} \div 30 = 7$$

$$7 \times 30 = \underline{\quad}$$

## **ESTIMATION**

Encourage estimations and checking answers throughout the year. Estimating is the ability to make reasonable guesses about a quantity. In the lower grades, learners deal with estimations informally and do not learn to round off.

Estimating is also very useful before starting a calculation. If the calculation is  $28 \times 42$ , learners should be taught to round the numbers to the nearest multiple of ten and get an estimate of the answer. For example,  $28 \times 42 \approx 30 \times 40 \approx 1\ 200$  (actual answer 1 176).

By the end of Grade 6 learners should be able to round off to 5, 10, 100, 1 000, 10 000, 100 000 and 1 000 000.

## **CONCLUSION**

The strategies listed above should be practised throughout the year in a structured mental mathematics programme. Learners can also play mathematical games to practise and memorise number facts. Calculators can be used to enhance the understanding of the strategies, but they should not be used during mental mathematics tests.

Mental mathematics is one of the most important tools for learning mathematics. It not only means to calculate quickly, but involves conceptual understanding and problem solving.

## **USEFUL WEBPAGES**

<http://www.compare4kids.co.uk/mental-maths.php>